Re-envisioning "good at math:" A case study of positive transformation

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ABSTRACT

Women account for 90% of US elementary teachers (Beilock, Gunderson, Ramirez & Levine, 2010), and many experience mathematics anxiety (Stoehr, 2017). Teachers can transmit this fear to students (Anderson, Boaler & Dieckmann, 2018). Mathematics teacher educators support teacher candidates in: assessing and overcoming fear of mathematics, reframing the teaching and learning of mathematics, and reframing what counts as mathematics. In its worst forms, mathematics anxiety rises to a level we refer to as math trauma--a state of debilitation when faced with doing mathematics. We have developed tools to determine:

- How do teachers view mathematics?
- How can we measure teachers' relationships with mathematics?
- How do these factors change over time?

In our case study, we report on changes for one prospective elementary teacher during two consecutive undergraduate mathematics courses. Marcy (pseudonym) began with a strong fear and dislike of mathematics, but eventually reported making peace with the subject. Many participants (N = 66) experienced similar trajectories, sharing that they needed a new relationship with mathematics to teach effectively. They were further motivated by new and positive experiences focused on collectively making sense of mathematics. Our qualitative case study, also in this special issue (Ruef, Runninghawk Johnson, Jacob, Jansen & Beavert 2020), shares Marcy's trajectory and evidence of the factors that motivated and informed the transformation in her relationship with mathematics. Our data include participants' drawings of "math" personified.

KEYWORDS

Education, Mathematics, Mathematics Anxiety, Transformation, Teachers

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Children in the United States of America begin their formal education in mathematics in elementary schools, where over 90% of their teachers are women (Beilock, Gunderson, Ramirez & Levine, 2010). Mathematics, as it is often socially constructed, can alienate women and minoritized people (Goins, 2017; Ruef, Jacob, Walker & Beavert, in review). Thus, it is unsurprising that many elementary school teachers are afraid of teaching mathematics (Ganley, Schoen, LaVenia & Tazaz, 2019; Stoehr, 2017). This fear can be transmitted to students (Anderson et al., 2018). For these reasons, mathematics teacher educators (MTEs) must concern themselves with helping preservice teachers (PTs) identify and overcome mathematics-related anxiety (Sweeny, Ruef, & Willingham, 2018).

Coursework for PTs can offer interventions to re-define what it means to do, teach, and learn mathematics. Teacher preparation classes can help PTs understand the roots of mathematics anxiety, heal from past experiences, and prepare to teach in ways that invite their future students to develop positive dispositions towards doing mathematics. Among the most powerful tools MTEs employ to address these concerns are sensemaking mathematical activities, the encouragement of risk-taking and productive struggle, and reframing mathematical success as deep conceptual understanding derived through productive discourse (AMTE, 2017; Barlow, Duncan, Lischka, Hartland & Willingham, 2017; Ruef, 2016; Ruef & Torres, in press). Many PTs need to see, do, and teach mathematics in ways that are very different from their past experiences as students. These are often new and therapeutic ways of knowing and doing mathematics.

As mathematics teacher educators and researchers, we developed and tested a suite of tools designed to determine the following:

- How do teachers view mathematics?
- How can we observe and interpret teachers' relationships with mathematics?
- How do teachers' views of and relationships with mathematics change over time?

We also teach classes that invite our students to identify and confront sources of mathematics anxiety and engage with mathematics in the ways we envision them teaching in the future (e.g AMTE, 2017; NCTM, 2014). Some PTs need opportunities to heal from past negative experiences they describe as debilitating. If mathematics anxiety regularly impedes a person's ability to do mathematics, it rises to a level we refer to as *math trauma*. People suffering from severe mathematics anxiety may exhibit physical symptoms such as raised heart rates, sweaty palms, and impaired ability to think (Finlayson, 2014). Beilock and colleagues (2010) found that when people attempt to do mathematics while enduring performance-related anxiety, their working memory is impaired. This makes it difficult, if not impossible, to compute or solve problems. Math anxiety is at best a distraction, at worst a disability.

As MTEs, our practice grows as we learn how coursework influences PTs' beliefs and envisioned future teaching practices. As an example of this line of inquiry, this case study reports on changes that occurred for one preservice teacher over the course of two consecutive undergraduate mathematics content courses.

METHODS

This case study draws from a larger multi-case study (Yin, 2017) involving 66 participants who were undergraduate students from two universities in the Pacific Northwest and Southwest of the United States. Based on pronoun preference, we identified 62 of the participants as female, three as male, and one as non-binary. Anecdotally, the demographics of these participants reflect the predominance of race and gender in the United States' elementary teaching force, which was 90% female and 80% White in 2017 (Loewus, 2017).

We collected data from a series of course activities, used to elicit participants' past and current experiences with mathematics, and their attributions of their successes, failures, and dispositions towards mathematics (Zazkis, 2015). Specifically, this data set includes the following coursework tasks: 1) draw a person who is "good at math," 2) write a short *mathography* describing past experiences with mathematics, 3) create a description of your personification of "Math" as a living thing, 4) draw that personification of Math, and 5) write a short script of a conversation you might have with your personification of Math. At the beginning of the second course, we repeated 6) the personification of mathematics drawing activity and added 7) a second script where the participant and Math meet for coffee and talk over their relationship. Finally, the PTs repeated the last two activities at the end of the second course. The chronology of this data collection is recorded in Table 1.

Table 1:Coursework Tasks and Dates Collected

Coursework Tasks	**Date Completed**
Draw a person who is "good at math"	January 19, 2017
Write a short mathography	January 19, 2017
Create a description of your personification of "Math"	January 21, 2017
Draw "Math"	January 21, 2017
Write a script of your conversation with "Math"	January 26, 2017
Draw "Math"	April 5, 2017
Write a script of your coffee chat with "Math"	April 7, 2017
Draw "Math"	June 5, 2017
Write a script of your coffee chat with "Math"	June 7, 2017

In our initial analysis of this data, we focused on a subset of our participants (n=12) and used a cycle of open and axial coding to thematically order the narrative portions of the data set and frame our interpretations of the participants' drawings. This analysis resulted in collections of trajectories describing our participants' relationships with mathematics and the factors to which they attributed changes in these relationships. In our analysis of the trajectories, we found similarities and patterns among the participants in our subset. Four of the participants described experiences with mathematics that were overall negative, while only one participant wrote about an overall positive relationship with mathematics. Three of the participants described experiences with mathematics that moved from positive to negative back to positive, so we categorized these as "Roller Coaster" trajectories. There were three participants who had complicated relationships with mathematics that didn't seem to fit in the other trajectories so we categorized them under the heading "It's Complicated." Finally, we had one participant who described their relationship with math as one that moved from distrust to trust. Table 2 provides an overview of the categories and groupings of participants.

Table 2: Participants' Trajectories in their Relationships with Mathematics (n=12)

Trajectory with Mathematics	Overall Negative	Overall Positive	Roller Coaster	It's Complicated	Distrust → Trust
Number of Participants	4	1	3	3	1

For our case study, we chose Marcy (a pseudonym), a White woman who was an undergraduate PT at the time of data collection. Marcy was typical of the participants who described an overall negative relationship with mathematics. Marcy was atypical in her poetic and verbose descriptions of the depth and breadth of her math anxiety. Coupled with her compelling drawings, Marcy's data described a journey of confrontation and change that echoed those of her classmates. Marcy's case was thus unique in its sharp definition and typical in its narrative trajectory from mathematical anxiety toward making peace with mathematics.

FINDINGS

We constructed the following narrative from samples of Marcy's two quarters of elementary mathematics content coursework. Marcy initially approached these classes with apprehension, writing in her mathography:

> My experiences with math have been pretty negative overall. I have one memory where we had a test coming up in my Algebra 2 class and I wasn't ready for it. So I went to all three after school review sessions my teacher had, worked with her one-on-one, and even went in for lunch to get help. I still failed the test. I've never been good at math, and I'm not a fan of it. I try to avoid doing it, and I've never looked forward to math class or mathematics in general. I just hope that I don't fail this class.
>
> -Marcy, January 19, 2017

Marcy's picture of someone who is good at math reflects a common theme of the growth mindset (Dweck, 2006) espoused by preservice teachers. She drew three different people and wrote, "Anyone can be good at math." However, Marcy did not appear to extend this attitude to herself, as evidenced by her written description of *who* Math was to her at that time. In her personification of Math dated January 21, 2017, she described it in these ways:

> Math looks like an F on a test. Math looks like counting sheep and missing one. Math looks like the humiliation of being the only one of your friend group to be a year behind in math and barely passing it. Math looks like sitting at the kitchen table with my father, trying to get through my homework without wanting to give up in frustration.

This theme was common throughout Marcy's writing--she repeatedly referenced the people, namely her father and teachers, who tried to help her, the efforts she made, and the inevitable failure she faced. Marcy rejected Math because Math rejected her. Poetically, she continued:

> Math is a faceless entity to me, one that is made of numbers and X's and Y's, and it's always hovering over my shoulder, like a circling vulture. It's never completely gone, though it sometimes gets bigger or smaller, lighter or darker. Math has been there since first grade, when I first told my teacher that "I don't understand" and that "I don't know the answer." It grew bigger as I went through middle school, and in high school this faceless entity was on my shoulder laughing at me as I got answer after answer wrong, and for the first time, failed a class.... But understanding math has always eluded me, and Math has just kept laughing at me from my shoulder (ellipsis added).

Once again Marcy mentioned failing, but with more impact--she failed a class. She also referenced her lack of understanding, re-framed as essential to learning mathematics in our PT courses. Marcy knew understanding was important and that she lacked it in the first grade. She described her dread growing as the stakes got higher. Marcy's personification of Math drawing, shown in Figure 1, captures her idea of a "a faceless entity . . . hovering over my shoulder, like a circling vulture."

Marcy's struggles with math centered on her negative prior experiences, and she appeared to be suffering a form of math trauma which consistently impeded her ability to engage with mathematics. But Marcy, like most of her classmates, had a powerful motivation to make peace with mathematics: She was studying to become a teacher and would be responsible for guiding her future students' relationships with mathematics. On January 26, 2017, in her conversation with Math script, she spoke directly to Math, saying: "Yea, but after this class is over I'll still need to 'get' you. I'll be a teacher for crying out loud, and if the teacher doesn't get math then how can my kids get it? I have to work with you, but I still don't get you."

Figure 1:Marcy's Personification of Math Drawing, January 19, 2017

On April 5, 2017, near the beginning of her second quarter in the content sequence, Marcy drew her next personification of Math, which she titled, the "Hammer of Math" (Figure 2). Notice that the head of the hammer is filled with detailed mathematical symbols, representations, and formulae. There is a long handle extending back to the right where we see a person wielding the hammer. A small figure, crushed under the shadow of the hammer, claws toward escape.

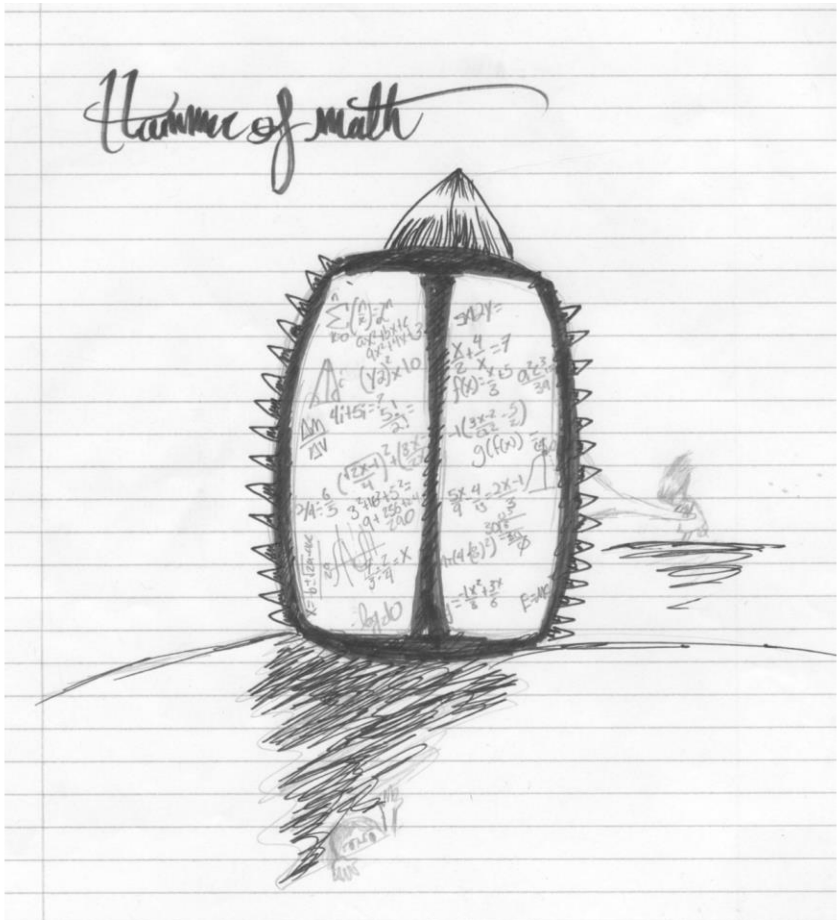

Figure 2: Marcy's second Personification of Mathematics Drawing, April 5, 2017

In the accompanying script for a coffee chat with the version of Math depicted in Figure 2, Math asks if Marcy is paying for the coffees. Marcy responds, "Considering our relationship, that's going to be a hard pass from me." Math has, of course, forgotten their wallet and Marcy begrudgingly pays for the drinks. It seems clear that Math is not living up to their end of the relationship--they start the conversation by pressing Marcy to pay for the drinks. During the course of their conversation, Marcy explains how mathematics has changed for her, and why it now matters in different ways.

> Well, math started to become more than just a list of problems for me to solve. I... it started to make sense. It was easier math too, don't get me wrong. I would probably still cry if you put calculus in front of me, but now... I actually have a fun math class. I got things right for the first time in a while. I didn't feel an impending sense of doom at the test (ellipses in original text).

Marcy once again referenced anxiety in test taking, which still appears to be central to her negative associations of mathematics. She goes on to say, "I have to at least act excited around you now that I'm planning to teach. My students have to at least feel neutral about math, and not be scared of it like I was." Once again Marcy referred to her envisioned future work as a teacher, this time describing her future students' dispositions towards mathematics.

Figure 3: Marcy's Third Personification of Mathematics, March, 2018

At the end of her two quarters in elementary mathematics content courses, Marcy once more drew her personification of Math (Figure 3). Marcy described Math as, "almost always the big, black fish; dark, menacing, scary. But sometimes, often when we least expect it, Math is the white fish; light, fun, shining bright, and friendly." This final drawing depicts a yin and yang symbol made of koi. It gives hope that Marcy was making peace with mathematics.

DISCUSSION

As researchers, this data is exciting and powerful. As MTEs, helping a student navigate such negative associations is a fraught and, at times, upsetting process. Empathy requires emotional engagement with our students. It is also challenging to disentangle potential data from teacher-pleasing responses. How can we tell if Marcy really changed her relationship with mathematics?

Anecdotally, Marcy demonstrated changes in the classroom. She volunteered to share her thinking more in class, and was willing to workshop incomplete ideas with input from the class. By the end of the second quarter, she wrote in a letter to her future self about the importance of using manipulatives, working with others, using diagnostic comments instead of points or letter grades, and avoiding "red pen, as it is scary." Marcy reminded herself to:

> Remember to teach math for understanding, not for the test. Give frequent opportunities for students to revise their work, since there is no learning tool more powerful and more valuable than learning from your mistakes. Math is hard and scary, but your job is make it less scary.

She added that it is important to, "Make mistakes in front of your students, and show how to recover from them with grace," and to, "emphasize, above all, that being confused is normal! Confusion does not mean you have failed. Confusion means that you are learning, and what you are looking for is not the right answer—it is learning." These beliefs were evident when Marcy volunteered to share her thinking in class. Like her classmates, she sometimes made mathematical errors and worked with the class to clarify and revise her arguments. To sum up for her future self, Marcy concluded:

> Finally, remember that there are no right answers. There are shades of gray, shades of who knows what color, and sometimes all you can do is try to match the shade to the situation. At first, you will be wrong. But keep trying, and through mistakes and failure, you will be more right.

In a separate letter, written to a future student in this course sequence, Marcy revealed more about her changed relationship with mathematics. She wrote that:

> I hated math. The very idea of someone asking me to do problem sets and make calculations was enough to make me grind my teeth. I walked into this class expecting a grueling quarter of doing problem sets, passing exams by the skin of my teeth, and homework painted red in errors. When I walked into this class, I expected two terms of slogging through numbers and two terms of feeling hopelessly incompetent.
>
> Instead, I got something very special. In this class, math became music. Numbers danced for me. Concepts that I had always known and never questioned were brought into new light, made alive with questions of why and how. My homework was lit up with smiley faces, my exams glowing with good scores. I asked my father, a long time math learner, “Is this what math is actually like?” Through the phone, I could hear him smile and say “Yes.”

CONCLUSION

Marcy's narrative is compelling. Woven through with themes of past trauma related to bad experiences with tests and lots of red ink, she made peace with mathematics by developing new relationships that invited color and dance into the ways she worked with mathematics. She moved away from seeing mathematics as a set of successes in test-taking and course grades and toward a vision of working with colleagues to make sense of mathematics, revising conjectures in a quest to understand mathematics. Marcy wrote eloquently of the new narrative she built for herself about taking risks, being wrong, being right, and most importantly, making sense of mathematics. Marcy's math trauma was easing back towards a more manageable level of math anxiety, leavened with new positive and affirming experiences.

Lest we overly congratulate ourselves, MTEs must remember that what happens in PT coursework does not always translate to their future teaching practices (Zeichner, Tabachnick & Densmore, 1987). Our future research goals include following PTs into their classrooms to see if they enact the vision of teaching they espoused during their studies.

This case study exemplifies many of the narratives we are constructing from the data provided by PT research participants. They partner with us in our work of engineering coursework that functions as a transformative experience. We invite our students to confront, engage with, and make peace with mathematics. It is true that not everyone has negative dispositions towards mathematics, but our data reflects that an overwhelming majority of our PTs do. Our work joins a growing body of research asking how we can make mathematics more open to more people by redefining what mathematics is, and what it means to be *good at math.*

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